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Computers and Mathematics with Applications 48 (2004) 1833–1846

An International Journal
**computers &
mathematics
with applications**

www.elsevier.com/locate/camwa

Existence of Positive Periodic Solutions of a Neutral Delay Lotka-Volterra System with Impulses

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(Received November 2003; accepted July 2004)

Abstract—Sufficient conditions are obtained for the existence of periodic positive solutions of a class of neutral impulsive delay Lotka-Volterra systems

$$N'_i(t) = N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) N_j(t - \tau_{ij}(t)) - \sum_{j=1}^n c_{ij}(t) N'_j(t - \gamma_{ij}(t)) \right],$$

$$N_i(t_k^+) - N_i(t_k) = b_{ik} N_i(t_k), \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots,$$

by using some techniques of Mawhin coincidence degree theory. My results generalize some known results. It is shown that under the appropriate linear periodic impulsive perturbations, the neutral impulsive delay Lotka-Volterra system preserves the original periodicity of the neutral nonimpulsive delay Lotka-Volterra system. © 2004 Elsevier Ltd. All rights reserved.

Keywords—Impulsive delay Lotka-Volterra system, Positive periodic solution, Coincidence degree.

1. INTRODUCTION AND PRELIMINARIES

A hallmark of observed population densities in the field is their oscillatory behavior. A main purpose of modeling population interactions is to understand what causes such fluctuations. There are four typical approaches for modeling such behavior.

- (i) Introduce more species into the model, and consider the higher dimensional systems (like predator-prey interactions [1,2]).
- (ii) Assume that the per capita growth function is time dependent and periodic in time [3].
- (iii) Take into account the time delay effect in the population dynamics [4].

Supported by the NNSF of China (10171040), the NSF of Gansu Province of China (ZS032-B25-030), the Foundation for University Key Teacher by the Ministry of Education of China, the Teaching and Research Award Program for Outstanding Young Teachers in Higher Education Institutions of Ministry of Education of China, the Key Research and Development Program for Outstanding Groups of Lanzhou University of Technology and the Development Program for Outstanding Young Teachers in Lanzhou University of Technology.

- (iv) Consider both the seasonality of the changing environment and the effects of time delays [1,5–8].

As Kuang [7] pointed out, Approach (i) is rather artificial, (ii) and (iii) emphasize only one aspect of reality; while (iv) is more realistic and interesting than (ii) and (iii).

On the other hand, there are some other perturbations in the real world such as fires and floods, that are not suitable to be considered continually. These perturbations bring sudden changes to the system. For example, consider the interaction between crops and local region. Once a year or every several years, a large amount of locusts may invade a region and cause damage to crops. This has been seen, often in recent years, in the northwestern province of China, Xingjiang and Inner Mongolia. Systems with such sudden perturbations involving impulsive differential equations have attracted the interest of many researchers in the past twenty years [9–19], since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, optimal control, etc. For details, see [9,11]. Recently, the corresponding theory for impulsive functional differential equations has been studied by many authors [10,12–19].

Since impulsive perturbations often make the systems more intractable except in some instances, the models can be rewritten as simple discrete-time mapping or difference equations when the corresponding continuous models can be solved explicitly. Most of the investigations related to impulsive systems are focused on the basic theory of impulsive equations and seldom give applications on biological systems. Naturally, more realistic and interesting models of populations should take into account (v) the impulsive effects, the seasonality of the changing environment and the effects of time delays.

The aim of this paper is to study the existence of positive periodic solution of the following neutral impulsive delay Lotka-Volterra system,

$$\begin{aligned} N'_i(t) &= N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) N_j(t - \tau_{ij}(t)) - \sum_{j=1}^n c_{ij}(t) N'_j(t - \gamma_{ij}(t)) \right], \\ N_i(t_k^+) - N_i(t_k) &= b_{ik} N_i(t_k), \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, \end{aligned} \quad (1.1)$$

where $\alpha_i(t)$, $\beta_{ij}(t)$, $\tau_{ij}(t)$, $\gamma_{ij}(t)$ are positive continuous periodic functions of period ω . For the ecological sense of system (1.1), I refer to [3] and the references cited therein. By using coincidence degree theory [20, p. 40], I shall establish some sufficient conditions for the existence of periodic positive solutions. My results indicate that under the appropriate linear periodic impulsive perturbations, the impulsive neutral delay Lotka-Volterra system (1.2) preserves the original periodicity of the neutral nonimpulsive delay Lotka-Volterra system,

$$\begin{aligned} N'_i(t) &= N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) N_j(t - \tau_{ij}(t)) - \sum_{j=1}^n c_{ij}(t) N'_j(t - \gamma_{ij}(t)) \right] \\ i &= 1, 2, \dots, n. \end{aligned} \quad (1.2)$$

That is to say, among the impulsive effects, the seasonality of the changing environment and the effects of time delays, the real generating or dominating force is intrinsic period.

Some special cases of neutral nonimpulsive delay Lotka-Volterra system (1.2) have been investigated. For example, Gopalsamy [21] has established the existence of a periodic solution for a periodic neutral delay logistic equation,

$$x'(t) = r(t)x(t) \left[1 - \frac{x(t - m\omega) + c(t)x'(t - m\omega)}{K(t)} \right], \quad (1.3)$$

where m is a positive integer, ω is a positive constant, $K, r, c \in C(R, R^+)$ are functions of period ω . Li [2] considered the following neutral nonimpulsive delay Lotka-Voterra system with constant delay of the form

$$N'_i(t) = N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) N_j(t - \tau_{ij}) - \sum_{j=1}^n c_{ij}(t) N'_j(t - \gamma_{ij}) \right], \quad (1.4)$$

$$i = 1, 2, \dots, n.$$

For equation (1.1), I shall make the following hypotheses.

- (H1) $0 < t_1 < t_2 < \dots$ are fixed impulsive points with $\lim_{k \rightarrow \infty} t_k = \infty$.
- (H2) $\{b_{ik}\}$ is a real sequence and $b_{ik} > -1$, $i = 1, 2, \dots, n$, $k = 1, 2, \dots$.
- (H3) $\prod_{0 < t_k < t} (1 + b_{ik})$, $i = 1, 2, \dots, n$, is periodic function of periodic ω .
- (H4) $\alpha_i(t)$, $\beta_{ij}(t)$, $\tau_{ij}(t)$, $\gamma_{ij}(t)$, $i, j = 1, 2, \dots, n$, are positive continuous periodic functions of periodic ω .

Here, and in the sequel, I assume that a product equals unity, if the number of factors is equal to zero.

I shall consider the solution of equation (1.1) with initial condition,

$$N_i(t) = \varphi_i(t), N'_i(t) = \varphi'_i(t), -\sigma \leq t \leq 0, \varphi_i(0) > 0, \quad (1.5)$$

$$i = 1, 2, \dots, n, \quad \varphi_i \in C([- \sigma, 0], [0, \infty)) \cap C^1([- \sigma, 0], [0, \infty)),$$

where $\sigma = \max_{1 \leq i \leq n, 1 \leq j \leq n} \{ \max_{t \in [0, \omega]} \{ \tau_{ij}(t), \gamma_{ij}(t) \} \}$.

DEFINITION 1. A function N_i , $i = 1, 2, \dots, n$, $\in ([-\sigma, \infty), [0, \infty))$ is said to be a solution of equation (1.1) on $[-\sigma, \infty)$ if the following are true.

- (i) $N_i(t)$ is absolutely continuous on each interval $(0, t_1]$ and $(t_k, t_{k+1}]$, $k = 1, 2, \dots$.
- (ii) For any t_k , $k = 1, 2, \dots$, $N_i(t_k^+)$ and $N_i(t_k^-)$ exist and $N_i(t_k^-) = N_i(t_k)$, $i = 1, 2, \dots, n$.
- (iii) $N_i(t)$ satisfies (1.1) for almost everywhere (a.e.) in $[0, \infty) \setminus \{t_k\}$ and satisfies $N_i(t_k^+) - N_i(t_k) = b_{ik} N_i(t_k)$, for every $t = t_k$, $i = 1, 2, \dots, n$, $k = 1, 2, \dots$.

Under the above hypotheses (H1)–(H4), I consider the neutral nonimpulsive Lotka-Voterra system

$$y'_i(t) = y_i(t) \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) y_i(t - \tau_{ij}(t)) - \sum_{j=1}^n C_{ij}(t) y'_j(t - \gamma_{ij}(t)) \right], \quad (1.6)$$

$$i = 1, 2, \dots, n.$$

with initial condition,

$$y_i(t) = \varphi_i(t), \quad y'_i(t) = \varphi'_i(t), \quad -\sigma \leq t \leq 0, \quad i = 1, 2, \dots, n, \quad (1.7)$$

$$\varphi_i(0) > 0, \quad i = 1, 2, \dots, n, \quad \varphi_i \in C([- \sigma, 0], [0, \infty)) \cap C^1([- \sigma, 0], [0, \infty)),$$

where

$$B_{ij}(t) = \prod_{0 < t_k < t - \tau_{ij}(t)} (1 + b_{ik}) \beta_{ij}(t), \quad C_{ij}(t) = \prod_{0 < t_k < t - \gamma_{ij}(t)} (1 + b_{ik}) c_{ij}(t). \quad (1.8)$$

By a solution $y_i(t)$, $i = 1, 2, \dots, n$, of (1.6) and (1.7), I mean an absolutely continuous function $y_i(t)$ $i = 1, 2, \dots, n$, defined on $[-\sigma, 0]$ that satisfies (1.6) a.e., for $t \geq 0$ and $y_i(t) = \varphi_i(t)$ $i = 1, 2, \dots, n$, on $[-\sigma, 0]$.

The following lemmas will be used in the proofs of our results. The proof of the first lemma is similar to that of Theorem 1 in [17].

LEMMA 1. Assume that (H1)–(H4) hold. Then,

- (i) if $y_i(t)$, $i = 1, 2, \dots, n$, is a solution of (1.6) on $[-\sigma, \infty]$, then, $N_i(t) = \prod_{0 < t_k < t} (1 + b_{ik}) y_i(t)$ $i = 1, 2, \dots, n$, is a solution of (1.1);
- (ii) if $N_i(t)$, $i = 1, 2, \dots, n$, is a solution of (1.1) on $[-\sigma, \infty]$, then, $y_i(t) = \prod_{0 < t_k < t} (1 + b_{ik})^{-1} N_i(t)$ $i = 1, 2, \dots, n$, is a solution of (1.6) on $[-\sigma, \infty]$.

PROOF. First, I prove (i). It is easy to see that $N_i(t) = \prod_{0 \leq t_k < t} (1 + b_{ik}) y_i(t)$, are absolutely continuous on the interval $(t_k, t_{k+1}]$ and, for any $t \neq t_k$, $k = 1, 2, \dots$,

$$\begin{aligned}
 & N'_i(t) - N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) N_j(t - \tau_{ij}(t)) - \sum_{j=1}^n c_{ij}(t) N'_j(t - \gamma_{ij}(t)) \right] \\
 &= \prod_{0 \leq t_k < t} (1 + b_{ik}) y'_i(t) \\
 &\quad - \prod_{0 \leq t_k < t} (1 + b_{ik}) y_i(t) \left[\alpha_i(t) - \sum_{j=1}^n \beta_{ij}(t) \prod_{0 \leq t_k < t - \tau_{ij}} (1 + b_{jk}) y_j(t - \tau_{ij}(t)) \right. \\
 &\quad \left. - \sum_{j=1}^n c_{ij}(t) \prod_{0 \leq t_k < t - \gamma_{ij}} (1 + b_{jk}) y_1(t - \gamma_{ij}(t)) \right] \\
 &= \prod_{0 \leq t_k < t} (1 + b_{ik}) \left[y'_i(t) - y_i(t) \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) y_i(t - \tau_{ij}(t)) - \sum_{j=1}^n C_{ij}(t) y'_i(t - \gamma_{ij}(t)) \right] \right] \\
 &= 0, \quad i = 1, 2, \dots, n.
 \end{aligned} \tag{1.9}$$

On the other hand, for every $t_k \in \{t_k\}$,

$$N_i(t_k^+) = \lim_{t \rightarrow t_k^+} \prod_{\sigma \leq t_j < t} (1 + b_{ij}) y_i(t) = \prod_{\sigma \leq t_j \leq t_k} (1 + b_{ij}) y_i(t_k), \quad i = 1, 2, \dots, n,$$

and

$$N_i(t_k) = \prod_{\sigma \leq t_j < t_k} (1 + b_{ij}) y_i(t_k), \quad i = 1, 2, \dots, n. \tag{1.10}$$

Thus, for every $k = 1, 2, \dots$,

$$N_i(t_k^+) = (1 + b_{ik}) y_i(t_k) \tag{1.11}$$

It follows from (1.9), (1.10), and (1.11) that $N_i(t)$, $i = 1, 2, \dots, n$, is the solution of (1.1).

Next, I prove (ii). Since $N_i(t)$, $i = 1, 2, \dots, n$, are absolutely continuous on each interval $(t_k, t_{k+1}]$ and, in view of (1.11), it follows that, for any $k = 1, 2, \dots$,

$$y_i(t_k^+) = \prod_{\sigma \leq t_j \leq t_k} (1 + b_{ij})^{-1} N_i(t_k^+) = \prod_{\sigma \leq t_j < t_k} (1 + b_{1j})^{-1} N_i(t_k) = y_i(t_k), \quad i = 1, 2, \dots, n,$$

and

$$y_i(t_k^-) = \prod_{\sigma \leq t_j \leq t_{k-1}} (1 + b_{ij})^{-1} y_i(t_k^-) = N_i(t_k), \quad k = 1, 2, \dots, \quad i = 1, 2, \dots, n,$$

which implies that $y_i(t)$, $i = 1, 2, \dots, n$, are continuous on $[\sigma, \infty)$. It is easy to prove that $y_i(t)$, $i = 1, 2, \dots, n$, are absolutely continuous on $[\sigma, \infty)$. Now, one can easily check that $y_i(t) = \prod_{0 < t_k < t} (1 + b_{ik})^{-1} N_i(t)$ $i = 1, 2, \dots, n$, is the solution of (1.6). The proof is complete.

LEMMA 2. (Mawhin [20, p. 40]). Let X and Z be two Banach spaces and L a Fredholm mapping of index zero. Assume that $N : \bar{\Omega} \rightarrow Z$ is L -compact on $\bar{\Omega}$ with Ω open bounded in X . Furthermore, assume:

(a) for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \text{Dom} L$,

$$Lx \neq \lambda Nx;$$

(b) for each $x \in \partial\Omega \cap \text{Ker} L$,

$$QNx \neq 0$$

and

$$\deg \{QNx, \Omega \cap \text{Ker} L, 0\} \neq 0.$$

Then $Lx = Nx$ has at least one solution in $\text{Dom } L \cap \bar{\Omega}$.

For convenience, I shall introduce the notation,

$$\bar{u} = \frac{1}{\omega} \int_0^\omega u(t) dt, \quad (u)_M = \max_{t \in [0, \omega]} |u(t)|, \quad (u)_m = \min_{t \in [0, \omega]} |u(t)|,$$

where u is a periodic continuous function with period ω .

2. MAIN RESULTS

Now, I state our first theorem for the existence of a positive ω -periodic solution of the equation (1.1).

THEOREM 2.1. Assume that (H1)–(H4) hold, and further assume the following:

(i)

$$\tau_{ij}(t) = \gamma_{ij}(t), \quad i, j = 1, 2, \dots, n,$$

(ii)

$$\tau_{ij}(t) \in C^2(R, [0, +\infty)), D_{ij}(t) = \frac{C_{ij}(t)}{1 - \tau'_{ij}(t)}, \quad v_{ij}(t) = \frac{B_{ij}(t) - D'_{ij}(t)}{1 - \tau'_{ij}(t)},$$

$$\tau'_{ij}(t) < 1, B_{ij}(t) - D'_{ij}(t) \geq 0$$

(iii)

$$\sum_{j=1}^n (C_{ij})_M e^{R_j} < 1,$$

(iv)

$$\sum_{j \neq i}^n (B_{ij}(t) - D'_{ij}(t)) e^{R_j} < \bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n;$$

(v) the system of the equations

$$\bar{\alpha}_i - \sum_{j=1}^n \bar{B}_{ij}(t) e^{u_j} = 0, \quad i = 1, 2, \dots, n,$$

has a unique solution $u^* = (u_1^*, u_2^*, \dots, u_n^*)^\top \in R^n$, where

$$R_i = \ln \frac{(\alpha_i)_M}{\theta_2 (v_{ii})_m} + \sum_{j=1}^n (D_{ij})_M \frac{(\alpha_i)_M}{\theta_1 (v_{ii})_m (1 - (\tau'_{ij})_M)} + 2\bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n,$$

where $B_{ij}(t)$, $C_{ij}(t)$ are defined by (1.8). Then, system (1.1) has at least one positive ω -periodic solution.

PROOF. Consider the system

$$x'_i(t) = \alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} - \sum_{j=1}^n C_{ij}(t) x'_j(t-\tau_{ij}(t)) e^{x_j(t-\tau_{ij}(t))}, \quad (2.1)$$

$$i = 1, 2, \dots, n.$$

In order to use Lemma 1 to equation (2.1), we take

$$X = \left\{ x(t) = (x_1(t), \dots, x_n(t))^T \in C^1(R, R^n) : x(t+\omega) = x(t) \right\}$$

and

$$Z = \left\{ z(t) = (z_1(t), \dots, z_n(t))^T \in C(R, R^n) : z(t+\omega) = z(t) \right\},$$

and denote $|x| = \sum_{i=1}^n |x_i|$, $|x|_\infty = \max_{t \in [0, \infty)} |x|$ and $\|x\| = |x|_\infty + |x'|_\infty$. Then, X and Z are Banach spaces when they are endowed with the norms $\|\cdot\|$ and $|\cdot|_\infty$, respectively. Set

$$N \begin{bmatrix} x_1 \\ x_n \end{bmatrix} = \begin{bmatrix} \alpha_1(t) - \sum_{j=1}^n B_{1j}(t) e^{x_j(t-\tau_{1j}(t))} - \sum_{j=1}^n C_{1j}(t) x'_j(t-\tau_{1j}(t)) e^{x_j(t-\tau_{1j}(t))} \\ \alpha_n(t) - \sum_{j=1}^n B_{nj}(t) e^{x_j(t-\tau_{nj}(t))} - \sum_{j=1}^n C_{nj}(t) x'_j(t-\tau_{nj}(t)) e^{x_j(t-\tau_{nj}(t))} \end{bmatrix}$$

and

$$Lx = x', \quad Px = \frac{1}{\omega} \int_0^\omega x(t) dt, \quad x \in X, \quad Qz = \frac{1}{\omega} \int_0^\omega z(t) dt, \quad z \in Z.$$

Evidently, $\text{Ker} L = \{x \mid x \in X, x = k \in R^n\}$, $\text{Im } L = \{z \mid z \in Z, \int_0^\omega z(t) dt = 0\}$ is closed in Z and $\dim \text{Ker} L = \text{codim Im } L = n$. Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \text{Im } L \rightarrow \text{Ker } P \cap \text{dom } L$ has the form

$$\begin{aligned} K_p(z) &= \int_0^t z(s) ds - \frac{1}{\omega} \int_0^\omega \int_0^t z(s) ds dt \\ &= \int_\omega^t z(s) ds + \frac{1}{\omega} \int_0^\omega sz(s) ds. \end{aligned}$$

Thus, by $D_{ij}(t) = C_{ij}(t)/1 - \tau'_{ij}(t)$, $i, j = 1, 2, \dots, n$, we have,

$$QN : X \rightarrow Z,$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\omega} \int_0^\omega \left[\alpha_1(t) - \sum_{j=1}^n (B_{1j}(t) - D'_{1j}(t)) e^{x_j(t-\tau_{1j}(t))} \right] dt \\ \vdots \\ \frac{1}{\omega} \int_0^\omega \left[\alpha_n(t) - \sum_{j=1}^n (B_{nj}(t) - D'_{nj}(t)) e^{x_j(t-\tau_{nj}(t))} \right] dt \end{bmatrix},$$

$$K_p(I - Q)N : X \rightarrow X,$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \int_\omega^t \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s) - D'_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} \right] ds \\ \vdots \\ \int_\omega^t \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s) - D'_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} \right] ds \end{bmatrix}$$

$$\begin{aligned}
& + \left[\frac{1}{\omega} \int_0^\omega s \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s) - D_{1j}(s) - sD'_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} \right] ds \right. \\
& \quad \vdots \\
& \quad \left. \frac{1}{\omega} \int_0^\omega s \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s) - D_{nj}(s) - sD'_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} \right] ds \right. \\
& \quad \quad \left. - \begin{bmatrix} \sum_{j=1}^n D_{1j}(t) e^{x_j(t-\tau_{1j}(t))} \\ \vdots \\ \sum_{j=1}^n D_{nj}(t) e^{x_j(t-\tau_{nj}(t))} \end{bmatrix} \right] \\
& + \left(\frac{1}{2} - \frac{t}{\omega} \right) \left[\int_0^\omega \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s) - D'_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} \right] ds \right. \\
& \quad \quad \quad \vdots \\
& \quad \quad \left. \int_0^\omega \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s) - D'_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} \right] ds \right].
\end{aligned}$$

Clearly, QN and $K_p(I-Q)N$ are continuous by the Lebesgue theorem and, moreover, $QN(\bar{\Omega})$, $K_p(I-Q)N(\bar{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Hence, N is L -compact on $\bar{\Omega}$. Here Ω is any open bounded set in X .

Corresponding to the equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\begin{aligned}
x'_i(t) &= \lambda \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} - \sum_{j=1}^n C_{ij}(t) x'_j(t-\tau_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} \right], \\
i &= 1, 2, \dots, n.
\end{aligned} \tag{2.2}$$

Suppose that $x(t) = (x_1, \dots, x_n) \in X$ is a solution of system (2.2) for a certain $\lambda \in (0, 1)$. By integrating (2.2) over the interval $[0, \omega]$, we obtain

$$\int_0^\omega \left[\alpha_i(t) - \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} \right] dt = 0, \quad i = 1, 2, \dots, n.$$

Hence,

$$\int_0^\omega \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt = \int_0^\omega \alpha_i(t) dt. \tag{2.3}$$

From the above and equation (2.2), we have

$$\begin{aligned}
& \int_0^\omega \left| \frac{d}{dt} \left[x_i(t) + \lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \right] \right| dt \\
&= \lambda \int_0^\omega \left| \alpha_i(t) - \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} \right| dt \\
&= 2\bar{\alpha}_i\omega, \quad i = 1, 2, \dots, n.
\end{aligned} \tag{2.4}$$

From (2.3), we obtain

$$\begin{aligned}
& \theta_1 \int_0^\omega \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt + \theta_2 \int_0^\omega \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt \\
&= \int_0^\omega \alpha_i(t) dt,
\end{aligned} \tag{2.5}$$

where $\theta_1, \theta_2 > 0$, and $\theta_1 + \theta_2 = 1$. Let $g_i = t - \tau_{ij}(t)$, $t = \varphi_{ij}(g_i)$ be the inverse function of $g_i = t - \tau_{ij}(t)$, then,

$$\begin{aligned} \int_0^\omega (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt &= \int_{-\tau_{ij}(0)}^{\omega-\tau_{ij}(\omega)} \frac{B_{ij}(\varphi_{ij}(g_i)) - D'_{ij}(\varphi_{ij}(g_i))}{1 - \tau'_{ij}(\varphi_{ij}(g_i))} e^{x_j(g_i)} dg_i \\ &= \int_0^\omega \frac{B_{ij}(\varphi_{ij}(g_i)) - D'_{ij}(\varphi_{ij}(g_i))}{1 - \tau'_{ij}(\varphi_{ij}(g_i))} e^{x_j(g_i)} dg_i. \end{aligned}$$

By using the mean value theorem of differential calculus, it implies that there exists $\eta_i \in [0, \omega]$ such that,

$$\int_0^\omega (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt = \frac{B_{ij}(\eta_i) - D'_{ij}(\eta_i)}{1 - \tau'_{ij}(\eta_i)} \int_0^\omega e^{x_j(t)} dt.$$

Since $v_{ij} = B_{ij}(t) - D'_{ij}(t)/1 - \tau'_{ij}(t)$, it follows (2.3) that,

$$\theta_1 \int_0^\omega \sum_{j=1}^n (B_{ij}(t) - D'_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt + \theta_2 \sum_{j=1}^n v_{ij}(\eta_i) \int_0^\omega e^{x_j(t)} dt = \int_0^\omega \alpha_i(t) dt. \quad (2.6)$$

By the mean value theorem of differential calculus, we see that there exists $\xi_i \in [0, \omega]$, such that,

$$\begin{aligned} \alpha_i(\xi_i) &= \theta_2 \sum_{j=1}^n v_{ij}(\eta_i) e^{x_j(\xi_i)} + \theta_1 \sum_{j=1}^n (B_{ij}(\xi_i) - D'_{ij}(\xi_i)) e^{x_j(\xi_i - \tau_{ij}(\xi_i))}, \\ i &= 1, 2, \dots, n, \end{aligned} \quad (2.7)$$

Hence,

$$x_i(\xi_i) < \ln \frac{\alpha_i(\xi_i)}{\theta_2 v_{ii}(\eta_i)} \leq \ln \frac{(\alpha_i)_M}{\theta_2 (v_{ii})_m}, \quad i = 1, 2, \dots, n. \quad (2.8)$$

and

$$e^{x_i(\xi_i - \tau_{ij}(\xi_i))} \leq \frac{\alpha_i(\xi_i)}{\theta_1 (B_{ij}(\xi_i) - D'_{ij}(\xi_i))} < \frac{(\alpha_i)_M}{\theta_1 (v_{ii})_m (1 - (\tau'_{ij})_M)}. \quad (2.9)$$

It follows from (2.4), (2.8), and (2.9) that,

$$\begin{aligned} x_i(t) + \lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} &\leq x_i(\xi_i) + \lambda \sum_{j=1}^n D_{ij}(\xi_i) e^{x_j(\xi_i - \tau_{ij}(\xi_i))} \\ &\quad + \int_0^\omega \left| \frac{d}{dt} \left[x_i(t) + \lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \right] \right| dt \\ &< \ln \frac{(\alpha_i)_M}{\theta_2 (v_{ii})_m} + \sum_{j=1}^n (D_{ij})_M \frac{(\alpha_i)_M}{\theta_1 (v_{ii})_m (1 - (\tau'_{ij})_M)} + 2\bar{\alpha}_i \omega \\ &= R_i, \quad i = 1, 2, \dots, n. \end{aligned}$$

As $\lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} > 0$, we obtain,

$$x_i(t) < R_i, \quad i = 1, 2, \dots, n. \quad (2.10)$$

Hence, we have

$$\begin{aligned} |x'_i(t)| &\leq \lambda \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} - \sum_{j=1}^n C_{ij}(t) x'_j(t - \tau_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} \right] \\ &< (\alpha_i)_M + \sum_{j=1}^n (B_{ij})_M e^{R_j} + \sum_{j=1}^n (C_{ij})_M (x'_j(t - \tau_{ij}(t)))_M e^{R_j}. \end{aligned}$$

According to Assumption (iii), we have

$$(|x'_i(t)|)_M < \frac{(\alpha_i)_M + \sum_{j=1}^n (B_{ij})_M e^{R_j}}{1 - \sum_{j=1}^n (C_{ij})_M e^{R_j}} = A_i, \quad i = 1, \dots, n. \quad (2.11)$$

By virtue of (2.3) and assumption (iv), it is easy to see that there exist points $\delta_i \in [0, \omega]$, $A'_i > 0$, such that,

$$|x_i(\delta_i)| < A'_i, \quad i = 1, 2, \dots, n.$$

By this and (2.11), we obtain,

$$|x_i(t)| < |x_i(\delta_i)| + \int_0^\omega |x'_i(t)| dt \leq A'_i + A_i \omega, \quad i = 1, 2, \dots, n.$$

Clearly, A'_i and A_i , $i = 1, 2, \dots, n$, are independent of λ . Denote $M = \sum_{i=1}^n (A'_i + A_i(1 + \omega)) + K$, where $K > 0$ is taken sufficiently large so that the unique solution $x^* = (x_1^*, x_2^*, \dots, x_n^*)^\top$ of the equation,

$$\bar{\alpha}_i - \sum_{j=1}^n B_{ij}(t) e^{x_j} = 0, \quad i = 1, 2, \dots, n,$$

satisfies $\|x^*\| < M$. Now, we take $\{\Omega = x(t) \in X : \|x\| < M\}$. This satisfies Condition (a) of Lemma 1.

When $x = (x_1, x_2, \dots, x_n)^\top \in \partial\Omega \cap \text{Ker} L = \partial\Omega \cap R^n$, x is a constant vector in R^n with $\|x\| = M$. Then,

$$QN \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \bar{\alpha}_1 - \sum_{j=1}^n \bar{B}_{1j} e^{x_j} \\ \vdots \\ \bar{\alpha}_n - \sum_{j=1}^n \bar{B}_{nj} e^{x_j} \end{bmatrix} \neq 0.$$

Furthermore, in view of Assumption (vi), it can easily be seen that

$$\deg \{QN x, \Omega \cap \text{Ker} L, 0\} \neq 0.$$

By now, we know that Ω verifies all the requirements of Lemma 1 and equation (2.1) has at least one ω -periodic solution. Then, by Lemma 1, we derive that equation (1.1) has at least one positive ω -periodic solution. The proof is complete.

In the assumptions of Theorem 1, I always require the condition $\tau_{ij}(t) = \gamma_{ij}(t)$, ($i, j = 1, 2, \dots, n$). In fact, if I assume that $C_{ij}(t) = C_{ij}$ ($i, j = 1, \dots, n$) are constants, then, I can drop Condition (i) of Theorem 1.

THEOREM 2.2. Assume that (H1)–(H4) hold, and further assume the following:

- (i) $C_{ij}(t) = C_{ij}$ ($i, j = 1, \dots, n$) are constants,
- (ii)

$$\tau_{ij}(t), \gamma_{ij}(t) \in C^2(R, [0, +\infty)), \quad \tilde{D}_{ij}(t) = \frac{C_{ij}}{1 - \gamma'_{ij}(t)}, \quad \bar{v}_{ij} = \frac{B_{ij}(t)}{1 - \tau'_{ij}(t)},$$

$$\gamma'_{ij}(t) < 1, \quad \tau'_{ij}(t) < 1, \quad \gamma''_{ij}(t) = 0.$$

(iii)

$$\sum_{j=1}^n C_{ij} e^{R'_j} < 1, \quad i = 1, 2, \dots, n,$$

(iv)

$$\sum_{j \neq i}^n (B_{ij}(t)) e^{R'_j} < \bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n; \quad i = 1, 2, \dots, n,$$

(v) the system of the equations

$$\bar{\alpha}_i - \sum_{j=1}^n \bar{B}_{ij}(t) e^{u_j} = 0, \quad i = 1, 2, \dots, n,$$

has a unique solution $u^* = (u_1^*, u_2^*, \dots, u_n^*)^\top \in R^n$, where

$$R'_i = \ln \frac{(\alpha_i)_M}{\theta_2 (\bar{v}_{ii})_m} + \sum_{j=1}^n (D_{ij})_M \frac{(\alpha_i)_M (1 - (\gamma'_{ij})_m)}{\theta_1 (\bar{v}_{ii})_m} + 2\bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n,$$

where $B_{ij}(t)$, $C_{ij}(t)$ are defined by (1.8). Then, system (1.1) has at least one positive ω -periodic solution.

PROOF. Set

$$N \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \alpha_1(t) - \sum_{j=1}^n B_{1j}(t) e^{x_j(t-\tau_{1j}(t))} - \sum_{j=1}^n C_{1j} x'_j(t - \gamma_{1j}(t)) e^{x_j(t-\gamma_{1j}(t))} \\ \vdots \\ \alpha_n(t) - \sum_{j=1}^n B_{nj}(t) e^{x_j(t-\tau_{nj}(t))} - \sum_{j=1}^n C_{nj} x'_j(t - \gamma_{nj}(t)) e^{x_j(t-\gamma_{nj}(t))} \end{bmatrix}$$

and $Lx = x'$, $Px = (1/\omega) \int_0^\omega x(t) dt$, $x \in X$, $Qz = (1/\omega) \int_0^\omega z(t) dt$, $z \in Z$, where X and Z are the same as those in the proof of Theorem 2.1. Thus, it is easy to see that,

$$QN : X \rightarrow Z$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega} \int_0^\omega \left[\alpha_1(t) - \sum_{j=1}^n B_{1j}(t) e^{x_j(t-\tau_{1j}(t))} - \sum_{j=1}^n C_{1j} x'_j(t - \gamma_{1j}(t)) e^{x_j(t-\gamma_{1j}(t))} \right] dt \\ \vdots \\ \frac{1}{\omega} \int_0^\omega \left[\alpha_n(t) - \sum_{j=1}^n B_{nj}(t) e^{x_j(t-\tau_{nj}(t))} - \sum_{j=1}^n C_{nj} x'_j(t - \gamma_{nj}(t)) e^{x_j(t-\gamma_{nj}(t))} \right] dt \end{bmatrix}.$$

By assumptions, and so $\tilde{D}'_{ij}(t) = 0$, it follows that

$$QN : X \rightarrow Z$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega} \int_0^\omega \left[\alpha_1(t) - \sum_{j=1}^n B_{1j}(t) e^{x_j(t-\tau_{1j}(t))} \right] dt \\ \vdots \\ \frac{1}{\omega} \int_0^\omega \left[\alpha_n(t) - \sum_{j=1}^n B_{nj}(t) e^{x_j(t-\tau_{nj}(t))} \right] dt \end{bmatrix}.$$

In which case, we can show that,

$$K_p(I - Q)N : X \rightarrow X,$$

$$\begin{aligned} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} &= \begin{bmatrix} \int_{\omega}^t \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} \right] ds \\ \vdots \\ \int_{\omega}^t \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} \right] ds \end{bmatrix} \\ &+ \begin{bmatrix} \int_0^{\omega} s \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} + \sum_{j=1}^n (\tilde{D}_{1j}(s)) e^{x_j(s-\gamma_{1j}(s))} \right] ds \\ \vdots \\ \int_0^{\omega} s \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} + \sum_{j=1}^n (\tilde{D}_{nj}(s)) e^{x_j(s-\gamma_{nj}(s))} \right] ds \end{bmatrix} \\ &+ \left(\frac{1}{2} - \frac{t}{\omega} \right) \begin{bmatrix} \int_0^{\omega} \left[\alpha_1(s) - \sum_{j=1}^n (B_{1j}(s)) e^{x_j(s-\tau_{1j}(s))} \right] ds \\ \vdots \\ \int_0^{\omega} \left[\alpha_n(s) - \sum_{j=1}^n (B_{nj}(s)) e^{x_j(s-\tau_{nj}(s))} \right] ds \end{bmatrix}. \end{aligned}$$

Therefore, for every bounded subset $\Omega \subset X$, N is L -compact on $\bar{\Omega}$. Corresponding to equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$x'_i(t) = \lambda \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} - \sum_{j=1}^n C_{ij}(t) x'_j(t - \gamma_{ij}(t)) e^{x_j(t-\gamma_{ij}(t))} \right], \quad (2.12)$$

$$i = 1, 2, \dots, n.$$

Suppose that $x(t) = (x_1, \dots, x_n) \in X$ is a solution of system (2.12) for a certain $\lambda \in (0, 1)$. By integrating (2.12) over the interval $[0, \omega]$, we obtain,

$$\int_0^{\omega} \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} - \sum_{j=1}^n C_{ij}(t) x'_j(t - \gamma_{ij}(t)) e^{x_j(t-\gamma_{ij}(t))} \right] dt = 0,$$

$$i = 1, 2, \dots, n.$$

Hence,

$$\int_0^{\omega} \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} dt = \bar{\alpha}_i \omega. \quad (2.13)$$

From the above and (2.12), we obtain

$$\begin{aligned} \int_0^{\omega} \left| \frac{d}{dt} \left[x_i(t) + \lambda \sum_{j=1}^n \tilde{D}_{ij}(t) e^{x_j(t-\gamma_{ij}(t))} \right] \right| dt &= \lambda \int_0^{\omega} \left[\alpha_i(t) - \sum_{j=1}^n B_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \right] dt \\ &< 2\bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n. \end{aligned}$$

From (2.13), we have

$$\theta_1 \int_0^{\omega} \sum_{j=1}^n (B_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt + \theta_2 \int_0^{\omega} \sum_{j=1}^n (B_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt = \int_0^{\omega} \alpha_i(t) dt. \quad (2.14)$$

By the same analysis as that in Theorem 2.1, we can get that there exists $\bar{\eta}_i \in [0, \omega]$ such that,

$$\int_0^\omega (B_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt = \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \int_0^\omega e^{x_j(t)} dt.$$

Since $\bar{v}_{ij} = B_{ij}(t)/1 - \tau'_{ij}(t)$, clearly,

$$\sum_{j=1}^n \bar{v}_{ij}(\bar{\eta}_i) \int_0^\omega e^{x_j(t)} dt = \int_0^\omega \alpha_i(t) dt$$

and

$$\begin{aligned} \int_0^\omega (B_{ij}(t)) e^{x_j(t-\tau_{ij}(t))} dt &= \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \int_0^\omega \frac{e^{x_j(t-\gamma_{ij}(t))}}{1 - \gamma'_{ij}(t)} dt \\ &= \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \frac{1}{1 - \gamma'_{ij}(\bar{\delta}_i)} \int_0^\omega e^{x_j(t-\gamma_{ij}(t))} dt, \end{aligned}$$

for some $\bar{\delta}_i \in [0, \omega]$. So,

$$\int_0^\omega \alpha_i(t) dt = \sum_{j=1}^n \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \frac{1}{1 - \gamma'_{ij}(\bar{\delta}_i)} \int_0^\omega e^{x_j(t-\gamma_{ij}(t))} dt. \quad (2.15)$$

It follows from (2.14) and (2.15) that

$$\int_0^\omega \alpha_i(t) dt = \theta_1 \sum_{j=1}^n \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \frac{1}{1 - \gamma'_{ij}(\bar{\delta}_i)} \int_0^\omega e^{x_j(t-\gamma_{ij}(t))} dt + \theta_2 \sum_{j=1}^n \bar{v}_{ij}(\bar{\eta}_i) \int_0^\omega e^{x_j(t)} dt.$$

That is,

$$\alpha_i(\bar{\zeta}_i) = \theta_1 \sum_{j=1}^n \frac{B_{ij}(\bar{\eta}_i)}{1 - \tau'_{ij}(\bar{\eta}_i)} \frac{1}{1 - \gamma'_{ij}(\bar{\delta}_i)} e^{x_j(\bar{\zeta}_i - \gamma_{ij}(\bar{\zeta}_i))} + \theta_2 \sum_{j=1}^n \bar{v}_{ij}(\bar{\eta}_i) e^{x_j(\bar{\zeta}_i)}.$$

Therefore, we have

$$\begin{aligned} x_i(\bar{\zeta}_i) &\leq \ln \left(\frac{(\alpha_i)_M}{\theta_2 (\bar{v}_{ii})_m} \right), \\ e^{x_i(\bar{\zeta}_i - \gamma_{ii}(\bar{\zeta}_i))} &< \frac{(\alpha_i)_M (1 - (\gamma'_{ij})_m)}{\theta_1 (\bar{v}_{ii})_m} \\ x_i(t) + \lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} &\leq x_i(\bar{\zeta}_i) + \lambda \sum_{j=1}^n D_{ij}(\bar{\zeta}_i) e^{x_j(\bar{\zeta}_i - \tau_{ij}(\bar{\zeta}_i))} \\ &\quad + \int_0^\omega \left| \frac{d}{dt} \left[x_i(t) + \lambda \sum_{j=1}^n D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \right] \right| dt \\ &< \ln \left(\frac{(\alpha_i)_M}{\theta_2 (\bar{v}_{ii})_m} \right) + \sum_{j=1}^n (D_{ij})_M \frac{(\alpha_i)_M (1 - (\gamma'_{ij})_m)}{\theta_1 (\bar{v}_{ii})_m} + 2\bar{\alpha}_i \omega \\ &= R'_i, \quad i = 1, 2, \dots, n. \end{aligned}$$

The rest of the proof is similar to the proof of Theorem 2.1 and it will be omitted. The proof of Theorem 2.2 is complete.

Similarly, if I further assume that $B_{ij}(t) = B_{ij}$ ($i, j = 1, \dots, n$) are constants, then, we have the following result.

THEOREM 2.3. Assume that (H1)–(H4) hold, and further assume the following:

- (i) $B_{ij}(t) = B_{ij}, C_{ij}(t) = C_{ij}$ ($i, j = 1, \dots, n$) are constants,
 (ii)

$$\tau_{ij}(t), \gamma_{ij}(t) \in C^2(R, [0, +\infty)), \quad \tilde{D}_{ij}(t) = \frac{C_{ij}}{1 - \gamma'_{ij}(t)}, \bar{v}_{ij} = \frac{B_{ij}}{1 - \tau'_{ij}(t)},$$

$$\gamma'_{ij}(t) < 1, \quad \tau'_{ij}(t) < 1, \quad \gamma''_{ij}(t) = 0.$$

(iii)

$$\sum_{j=1}^n C_{ij} e^{R_j''} < 1, \quad i = 1, 2, \dots, n,$$

(iv) the system of the equations,

$$\bar{\alpha}_i - \sum_{j=1}^n B_{ij} e^{u_j} = 0, \quad i = 1, 2, \dots, n,$$

has a unique solution $u^* = (u_1^*, u_2^*, \dots, u_n^*)^\top \in R^n$, where,

$$R_i'' = \ln \left(\frac{(\alpha_i)_M}{\theta_2 (\bar{v}_{ii})_m} \right) + \sum_{j=1}^n (D_{ij})_M \frac{(\alpha_i)_M (1 - (\gamma'_{ij})_m)}{\theta_1 (\bar{v}_{ii})_m} + 2\bar{\alpha}_i \omega, \quad i = 1, 2, \dots, n,$$

where $B_{ij}(t), C_{ij}(t)$ are defined by (1.8). Then, the system (1.1) has at least one positive ω -periodic solution.

The proof is similar to the proof of Theorem 2.2 and will be omitted.

REMARK 1. My results in this paper indicate that under the appropriate linear periodic impulsive perturbations, the impulsive delay Lotka-Volterra system (1.1) remains the original periodicity and global attractivity of the neutral impulsive delay Lotka-Volterra system (1.2).

REMARK 2. If we let $\theta_1 = \theta_2 = 1/2$, $\tau_{ij}(t) = \tau_{ij}$, $\gamma_{ij}(t) = \gamma_{ij}$ and consider the following neutral nonimpulsive delay Lotka-Volterra system.

$$N'_i(t) = N_i(t) \left[\alpha_i(t) - \sum_{j=1}^n b_{ij}(t) N_i(t - \tau_{ij}) - \sum_{j=1}^n c_{ij}(t) N'_i(t - \gamma_{ij}) \right]. \quad (2.16)$$

By employing Theorems 2.1–2.3, we obtained sufficient conditions for the existence of positive periodic solutions of (2.16), which has been proved in [2]. So our results generalized the main results in [2].

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